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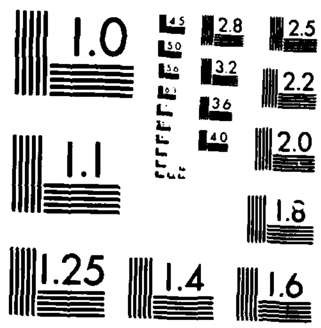

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A MODIFIED KOLMOGOROV-SMIRNOV, ANDERSON-DARLING, AND CRAMER-VON MISES TEST FOR THE CAUCHY DISTRIBUTION WITH UNKNOWN LOCATION AND SCALE PARAMETERS

THESIS

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Space Operations

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Frank Ocasio, B.S., M.S.
Captain, USAF

December 1985

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Preface

The purpose of this study was to produce a set of critical value tables for the Cauchy distribution using three popular goodness-of-fit tests, the Kolmogorov-Smirnov, the Anderson-Darling, and the Cramer-von Mises. This will allow anyone doing hypothesis testing to test a null hypothesis involving the Cauchy. To determine the confidence the user may have when using these tables, a power comparison was run against several alternate distributions.

When preparing this thesis, I received a great deal of help and support from others. My faculty advisor, Dr. A.H. Moore, helped keep me within the original scope of the thesis effort, which made it possible to finish on time. Capt. Jim Porter was very helpful in finalizing my computer programs, and without his help I would still be working on those programs. My two little boys, Mike and Matt, helped me by maintaining my overall perspective, and providing me enough breaks to maintain my sanity. Finally, my wife Kellie deserves more thanks than she will probably ever get as she supported me through my numerous long nights during the thesis preparation.

Frank Ocasio

Table of Contents

Abstract

The Kolmogorov-Smirnov, Anderson-Darling, and Cramer-von Mises critical values are generated for the Cauchy distribution. The critical values are used for testing the null hypothesis that a set of observations follow a Cauchy distribution when the location and scale parameters are unknown and estimated from the sample. A Monte Carlo simulation, using 5000 repititions, was used to generate the critical values for sample sizes of 5(5)30 and 50.

A power study was performed using Monte Carlo simulation for the Kolmogorov-Smirnov, Anderson-Darling, and Cramer-von Mises tests. Sample sizes of 5, 15, 25, and 50 were used for six alternate distributions, for alpha levels of .05 and .01. Analyzing by sample size shows very poor power for a sample size of five. As the sample size increases so does the power, so that at a sample size of fifty, the powers against three of the six distributions is .5 or better. Among the three tests, the Kolmogorov-Smirnov is consistently more powerful, regardless of sample size or alpha level.

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I. Introduction

Chapter Overview

This chapter gives an outline of the scope of this thesis. Some background will be covered on data analysis and modeling, tying that into goodness-of-fit testing. Then the Problem Statement, the Research Question, and the Research Objectives will be given.

Background

When data are being analyzed, one of the first things to do is develop a valid model of that data. This is a four step process (5:332), with the first being data collection. The next step is to analyze the empirical data distribution and attempt to match it against a known distribution. This is done using a histogram, which gives a visual image of the data distribution. Third, the parameters of that known distribution, most often location and

scale, are estimated from the data. A familiar example of these parameters is the mean and variance of the Normal distribution. The fourth step is to apply goodness- of-fit tests. Here a null hypothesis (H_0) is proposed which states that the actual distribution of the data is the known distribution, whereas the alternate hypothesis (H_1) is that the actual distribution is not the known distribution. The tests measure the fit between the empirical and known distributions. To use the tests, statistics are calculated from the data and compared to critical value tables which have been developed for various distributions. The comparison will result in accepting or rejecting H_0. If H_0 is rejected, the process is repeated, starting with the second step.

The three goodness-of-fit tests used for this thesis apply different techniques to determine fit. The Kolmogorov-Smirnov (KS) test uses the absolute difference between the empirical and known distributions. A problem with the KS test is that it tends to have smaller discrepencies at the tails rather than near the median of the distribution (39:6). One way to overcome this problem is to use the squared differences between the distributions. The Anderson-Darling (AD) test uses a weighted squared difference and the Cramer-von Mises (CVM) uses only the squared difference.

This thesis will look at the Cauchy distribution. It is similar in shape to the Normal except that it has longer and flatter tails (21:154). In physics,

the Cauchy is used in modeling Brownian motion (32:161).

Problem Statement

Highly accurate goodness-of-fit tests have not been developed for the Cauchy distribution with unknown location and scale parameters. These tests would require critical value tables based on the data sample size and parameters.

Research Question

How can the KS, AD, and CVM tests be modified for the Cauchy distribution when the location and scale parameters are unknown?

Research Objectives

1. Generate and document critical value tables for the modified Kolmogorov-Smirnov, Anderson-Darling, and Cramer-von Mises goodness-of-fit tests.
2. Do a power study of the Kolmogorov-Smirnov, Anderson-Darling, and Cramer-von Mises tests to determine the most powerful. The power is the probability of rejecting H_0 when H_1 is true (6:79). The higher the power, the greater the confidence in the test results.

II. Goodness of Fit Tests

Chapter Overview

This chapter will develop the background for goodness-of-fit (GOF) tests. First, hypothesis testing will be covered as an introduction to GOF. This will be followed by a look at GOF tests. The X^2 will be covered as the most common of these tests. Then the concept of the empirical distribution function (EDF), and its use in GOF, will be discussed. Finally, the EDF tests which will be used in this thesis, the Kolmogorov-Smirnov (KS), the Anderson-Darling (AD) and the Cramer-von Mises (CVM), will be introduced.

Hypothesis Testing

In hypothesis testing, a specific statement (called the hypothesis) is made about a population. Then a sample is taken from that population. Based on that sample, a decision is made for or against accepting the hypothesis (7:75). That decision is based on the following test procedure (7:75-77):

1. A hypothesis to be tested, the null hypotheses (H_0), is made about the population. The negative of H_0 is also set up and labelled H_1.
2. To make the decision, a test statistic is used. This statistic

would assign real numbers to points in the sample space and allow ordering of those points based on their ability to tell the difference between a true and a false H_0.

3. A rule is established to determine which values of the statistic will allow acceptance and which rejection. For this thesis, larger values of the statistic tend toward rejection of H_0. That value of the test statistic which is the cutoff between accepting and rejecting is called the critical value, and if the test statistic is greater than that value, H_0 is rejected.

4. A random sample is taken from the population. Based on that sample the test statistic is evaluated, and the hypothesis is then either accepted or rejected.

The sample that is taken is only part of the population, and therefore contains only part of the total information available. This leads to a possibility of error when deciding whether to accept or reject H_0. This error can surface in two ways: H_0 can be rejected when it is actually true, which is called a Type I error; H_0 can be accepted when it is actually false, which is called a Type II error (7:78). Since hypothesis testing is concerned with minimizing these errors, the maximum probabilities of making these errors

have been given the labels of α for Type I and β for Type II. Related to β is the parameter of power, or $1-\beta$, the probability of rejecting H_0 when false.

The basic thrust behind hypothesis testing is to reject H_0, while with GOF testing the reverse is true (1:72).

GOF Tests

H_0 in GOF tests is that a selected distribution fits the distribution underlying the population sample. One common way to get that selected distribution is to plot the sample data points using a histogram and pick a distribution that visually matches that histogram.

GOF tests try to determine if there is any *evidence of disagreement* between the sample and the selected distribution (1:72). The assumption is that the sample data fits the distribution unless there is enough evidence to disprove that assumption. An intuitive approach to collecting the evidence is to first plot the sample distribution function:

$$F_n(X) : r/n \tag{1}$$

where r=number of $x_i \leq x$. Then compare $F_n(X)$ with the assumed distribution, and visually inspect for substantial disagreement (11:290). However, to

attain accurate, reproduceable results some standard is required to measure the discrepancy. This is where the GOF tests come in.

The best-known GOF test is the Chi-Square (1:73). The test first groups the sample data into classes then compares the observed frequency of $F_n(X)$ in each of the classes with the expected frequency of the assumed distribution (39:2). The test statistic is (1:73):

$$X^2 = \sum_{i=1}^{k} [(f_{o_i} - f_{e_i})^2 (f_{e_i})^{-1}] \tag{2}$$

where

f_{o_i} = the observed frequency per class

f_{e_i} = the expected frequency per class

k = the number of classes

Some of the advantages with this test are it is good for a discrete distribution and the statistic can be adjusted if the parameters of $F_n(X)$ are estimated from the sample (33:731). A disadvantage is that the sample sizes must be fairly large ($n > 25$) for the test to work. This minimum n is to allow sufficient data points in each class to calculate the test statistic (1:73).

Another set of GOF tests use statistics based on the sample, or

empirical, distribution function, otherwise known as EDF statistics (33:732). With these tests, a comparison is made between $F_n(X)$ (the EDF), and F(X), the assumed cumulative distribution function (CDF), to see if they match (35:1). $F_n(X)$ is defined above, where the n values of x_i are a random sample from X.

From the x_i, if $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ are set up as ascending ordered statistics, then $F_n(X)$ is defined by (35:1)

$$F_n(X) = 0 \qquad x < x_{(1)} \tag{3}$$

$$F_n(X) = i/n \qquad x_i \leq x < x_{(i+1)},\ i = 1, \ldots, (n-1) \tag{4}$$

$$F_n(X) = 1 \qquad x_{(n)} < x. \tag{5}$$

The expectation is that $F_n(X)$, the proportion of the random sample < x, would give a good estimate of F(X), the probability of X < x, which it does (35:1). This leads to the development of the EDF statistics which use the discrepancy between $F_n(X)$ and F(X) to determine if the sample comes from F(X).

Some advantages with using EDF statistics are that, unlike the Chi-Square, they can be used with small sample sizes, and, when F(X) is fully specified, they are more powerful than the Chi-Square test (33:732). One disadvantage is that EDF statistics cannot be used for discrete distributions.

Another disagvantage was that, initially, EDF statistics coulo only be

used when F(X) was fully specified. This was due to the use of the probability integral transformation, which, when used with a fully specified CDF, will convert the values of that CDF to ordered values from zero to one based on a uniform distribution (39:5). If the parameters of F(X) were estimated, the cumulative distribution of the EDF statistics would depend not only on the sample size, but also on the value of the unknown parameters (33:731). This limitation to a fully specified F(X) prevented the widespread use of EDF statistics, since the parameters of the assumed distribution are usually not known beforehand and must be estimated from the sample data.

In 1948, David and Johnson (10) changed that when they showed that if invariant estimates of only the location and scale parameters are taken from the sample data, then the cumulative distribution of the EDF statistics will depend on the functional form of F(X), not on the estimated parameters. This cleared the way for modified (using estimated parameters) tables of critical values to be generated for a variety of distributions which would depend only on sample size and significance level (α). The first was H. W. Lilliefors for the normal (25) and exponential distribution (26). J. G. Bush did tables for the Weibull distribution (6), and P. J. Viviano did so for the Gamma distribution (36). Green and Hegazy did tables for the Uniform, Normal, Laplace, Exponential and Cauchy distributions (16). This thesis will do a new set of critical value tables for the Cauchy because Green and Hegazy did not use the

same estimating technique for parameter estimation, and they did not use the bootstrap interpolation technique, which will be discussed in Chapter IV.

As with all the GOF tests, the intent in hypothesis testing when using EDF statistics is to accept H_0. This can make power problems a significant concern. The desire would be for the results of the testing to be powerful, i.e., to accept H_0 and also feel confident that the alternate hypothesis is false. However, this is not always the case. One problem is that though EDF statistics can be used with small sample sizes, the results are not very powerful (29:3). For example when Green and Hegazy did power studies on their statistics, for n = 5, the power was never greater than 0.5 (16). Another problem is that the statistics are more powerful against some distributions (*21:3). This makes the results of power studies helpful to anyone using these statistics, since, assuming H_0 is accepted, the power study can be referenced to determine how much confidence can be had in the results.

EDF Statistics

This thesis will work with three EDF statistics, the Kolmogorov-Smirnov statistic (KS), the Cramer-von Mises statistic (CVM), and the Anderson-Darling statistic (AD).

The KS statistic is defined as (*21:15):

$$D = \max |F^*(x) - F(X)| \qquad (6)$$

where

the x_i are ordered

$F^*(x)$ is the CDF value of the data point

It is based on the greatest vertical difference between the two functions (35:2), and has these advantages over the Chi-Square (28:76):

- It does not lose information by grouping whereas the Chi-Square does, and this information loss is large for small samples, making the KS statistic a better choice for small samples.
- The KS statistic is easier to determine computationally.

One problem with the KS is its insensitivity to differences in the tails, since both functions tend to 0 and 1 in those extremes (12:3).

A more flexible set of statistics is the Cramer-von Mises family, to which both of the other statistics belong. This family incorporates a weight function, $\Psi(X)$, which allows weighting the deviations based on the importance of different portions of the distribution function (2:194). These statistics are based on the integral of the weighted squared difference between the assumed distribution and the EDF (35:2):

$$W^* = \int_{-\infty}^{\infty} [F_n(X) - F(X)]^2 \, \Psi(X) \, dx \qquad (7)$$

The CVM statistic is W^* with $\Psi(X) = 1$ (35:2). The computational form of this statistic is (16:205):

$$W^2 = (12n)^{-1} + \sum_{i=1}^{n}[Y_i - (2i - 1)(2n)^{-1}]^2 \qquad (8)$$

where $Y_i = F(x_i)$.

The AD statistic sets the weight function equal to the inverse of the variance of F(X) (35:2):

$$\Psi[F(X)] = [\{F(X)\}\{1 - F(X)\}]^{-1} \qquad (9)$$

This assigns equal weights to each point of F(X) (2:195), increasing the weight given to the tails of the distribution, and providing better detection of differences in the tails than the KS or the CVM statistics (34:360).

The computational form of this statistic is (16:206):

$$A = -n - (n)^{-1}\{\sum_{i=1}^{n}(2i - 1)[\ln Y_i + \ln(1 - Y_{n+1-i})]\} \qquad (10)$$

III. The Cauchy Distribution

Chapter Overview

This chapter will discuss various aspects of the Cauchy distribution. The first aspect is the definition of the distribution, covering the pertinent equations. Then the properties of the Cauchy are covered, followed by a brief glance at some of its uses. Finally, parameter estimation for the Cauchy will be discussed, from a general look at estimation to a discussion of the estimation technique to be used in this thesis.

Definition

The Cauchy probability density function (PDF) is (21:154):

$$(\pi\lambda)^{-1}\{1+[(x-\theta)/\lambda]^2\}^{-1} \qquad (11)$$

where

$\lambda > 0$

λ is the scale parameter

θ is the location parameter

The CDF for the Cauchy is (17:404):

$$\tfrac{1}{2} + \pi^{-1}\tan^{-1}[(x - \theta)/\lambda] \qquad (12)$$

The characteristic function is (22:11):

$$C_x(t) = \exp(it\theta - |t|\lambda) \qquad (13)$$

The K^{th} partial derivative of $C_x(t)/i^k$ with respect to t, when evaluated at t = 0, is the K^{th} moment (22:11).

<u>Properties of the Cauchy</u>

Given $C_x(t)$, an evaluation of the first partial derivative with respect to t at t = 0 yields an imaginary solution, resulting in all higher partials being imaginary (22:11). This leads to an oddity of the Cauchy, namely, that it has no moments of order ≥ 1, and therefore has an infinite expected value and standard deviation (21:154).

Though it has no finite expected value, the Cauchy is symmetric about its expected value, and is a member of the symmetric stable family (24:133). This symmetry is similar to the Normal, except for the longer and flatter tails of the Cauchy (21:154).

There are some other properties to note (30:303-305):

1. The distribution of the reciprocal of a Cauchy variable is the same as that of the variable.
2. The arithmetic means of samples from the Cauchy have the same distribution as the Cauchy.
3. The distribution of the product and quotient of two Cauchy variables is:

$$f(u) = [\pi^2(u^2 - 1)]^{-1} \log (u^2) \tag{14}$$

<u>Uses for the Cauchy</u>

As a member of the symmetrical stable family of distributions, the Cauchy has applications in economic modeling and estimation (15:275). Time-series and cross-section data for such things as personal incomes, stock and commodity price changes, and employment measures of businesses often were assumed to behave as normally distributed random variables. However, frequency functions consistently came up with too much mass in the tails to be accounted for by the normal. The Cauchy, with its longer and flatter tails, allows for that mass. This backs up the statement made by Haas and Bain that "the Cauchy distribution should be considered as a possible model whenever one needs a density function with heavier tails than the normal distribution allows" (17:403).

Fig. 1 is a geometrical application of the Cauchy distribution (21:161).

In this model the Cauchy distribution represents the distribution of P, the point of intersection of a variable straight line with a fixed straight line. The variable straight line is randomly oriented in two dimensions through the fixed point A. The result is the distance OP is Cauchy distributed with $\theta = 0$.

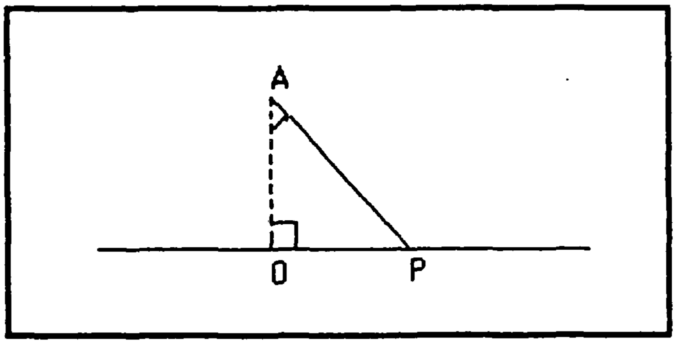

Fig 1. Cauchy Distribution Model (21:161)

Using this model, the Cauchy distribution can represent the distribution of points where particles from a point source, shown as A, impact a fixed straight line (21:161). This is used in physics, where the Cauchy distribution is used to help describe the motion of a random point in standard Brownian motion (32).

Estimation

Estimation is a procedure that allows generalizing from a sample to a population (37:334). In this thesis the concern is with point estimation,

where a sample statistic is used to estimate a population parameter. There are several desirable properties for point estimators (37:335-342):

1. Unbiasedness: where the expected value of the estimator (G) is equal to the parameter (θ), i.e., $E(G) = \theta$.
2. Consistency: where the larger the sample, the higher the probability of G being close to θ.
3. Relative Efficiency: that the estimator be more efficient (smaller α) than other estimators.
4. Sufficiency: that the estimator contain all the information available in the data about the parameter.

One method of estimation uses the sample as the guide to the parameter (37:345). With sample values $(x_1, x_2, \ldots, x_n)$, a likelihood function is set up:

$$L(x_1, \ldots, x_n \mid \theta) \tag{15}$$

This is the likelihood of getting this particular sample, given some θ. The maximum likelihood principle says to take as an estimate of θ that value which, while within the range of θ, makes the likelihood function as large as possible (23:35). For computational purposes, it is usually easier to work with log L.

An attractive feature of the maximum likelihood estimator (MLE) is that

it is invariant (37:346). Invariance, in terms of the variables used above, means that if G is the MLE of θ and $h(\theta)$ has an inverse, then $h(G)$ is an MLE of $h(\theta)$. An example is with a sample taken from a normally distributed population. For this case S^2, the sample variance, is an MLE of σ^2, the population variance. Invariance says that the sample standard deviation, S, is also an MLE of the population standard deviation, σ. The invariance of the MLE is important for this thesis, since invariant estimators of the location and scale parameters are needed to develop critical value tables when F(X), the hypothesized distribution, is not fully specified.

Applied to the Cauchy distribution, the likelihood function is (17:404):

$$L(x_1, \ldots, x_n \mid \theta, \lambda) = \prod_{1}^{N} \{\pi\lambda[1 + (x_i - \theta/\lambda)^2]\}^{-1} \tag{16}$$

and the maximum likelihood equations are:

$$\sum_{i=1}^{n} \{ [(x_i - \hat{\theta})\, \hat{\lambda}^{-1}] [1 + (x_i - \hat{\theta})^2\, \hat{\lambda}^{-2}]^{-1} \} = 0 \tag{17}$$

$$\sum_{i=1}^{n} [1 + (x_i - \hat{\theta})^2\, \hat{\lambda}^{-2}]^{-1} = \tfrac{1}{2} n \tag{18}$$

where $\hat{\theta}$ and $\hat{\lambda}$ are the MLE for θ and λ, respectively. These equations are then solved for $\hat{\theta}$ and $\hat{\lambda}$.

The MLE is not the only estimation technique that could have been used for this thesis. Another popular estimator is the BLUE, or best linear unbiased estimator. However, a study by Haas, Bain, and Antle (17) concluded that the MLE is a better estimator for the Cauchy distribution,since they found the confidence intervals developed for the parameters were narrower with the MLE than with the BLUE.

IV. Methodology

Chapter Overview

This chapter looks at the methodology used to complete this thesis. The specific steps used in the Monte Carlo method to generate the critical value tables will be looked at first, followed by a discussion of the steps used to do the power study.

Generating the Critical Value Tables

This thesis used the Monte Carlo method to generate critical value tables for the Cauchy distribution. This method is a way to investigate the behavior of probabalistic processes. It takes random numbers, chosen so that they simulate the properties of the process being investigated, and observes their behavior, from which conclusions can be drawn about that process (18:2-4).

Fig. 2 is a flow chart showing the logic for generating the critical value tables (6:13-14). The following discussion will elaborate on those steps:

Step 1: Random Deviate Generation. To start the Monte Carlo analysis, random Cauchy deviates need to be attained. A commercially available computer subroutine, GGCAY, was used to generate those deviates. It is part of the International Mathematical and Statistic Library (IMSL) (20:Chapt G).

Step 2: Ordering the Random Deviates. Another IMSL subroutine, VSRTA,

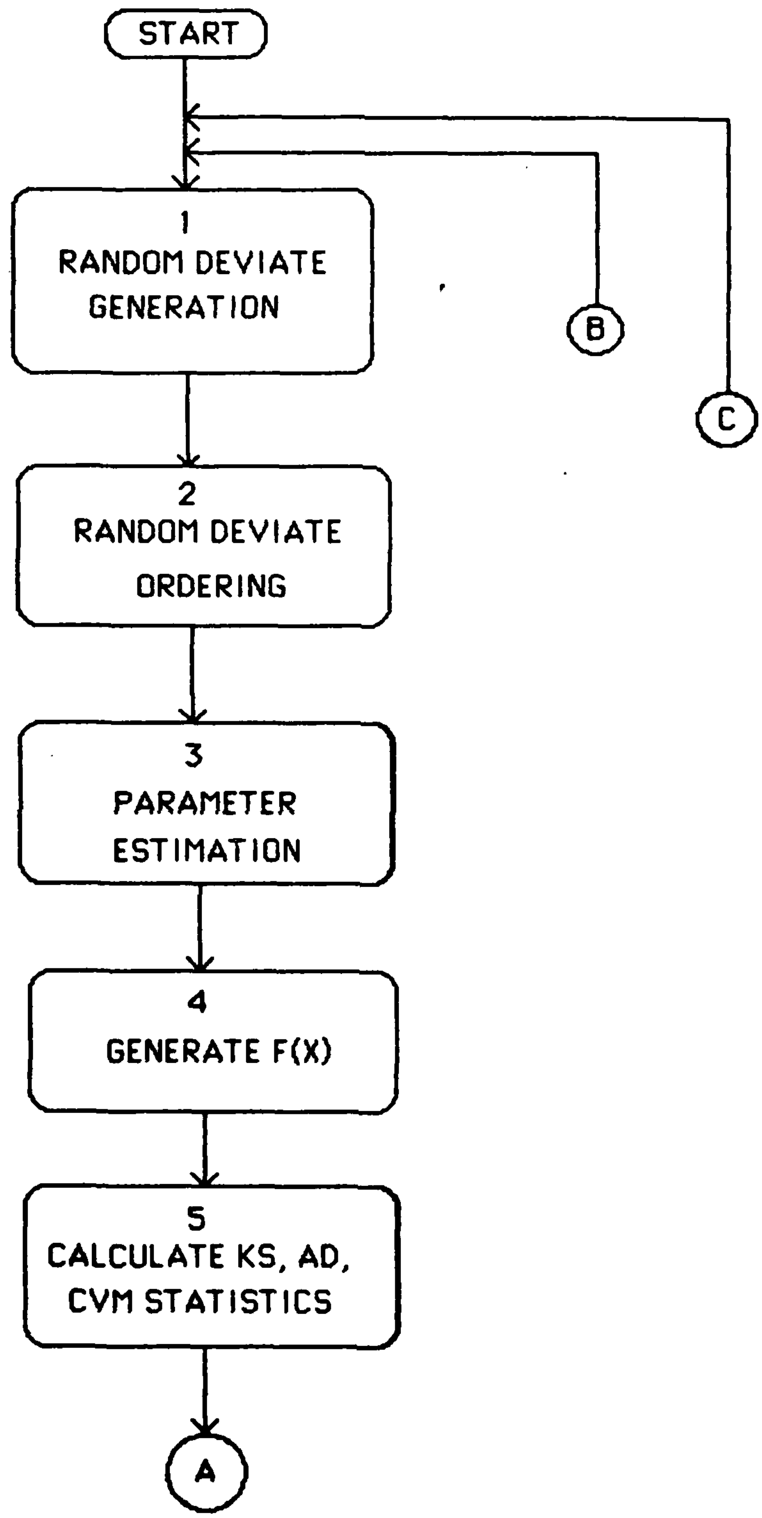

Fig. 2 Critical Value Flowchart

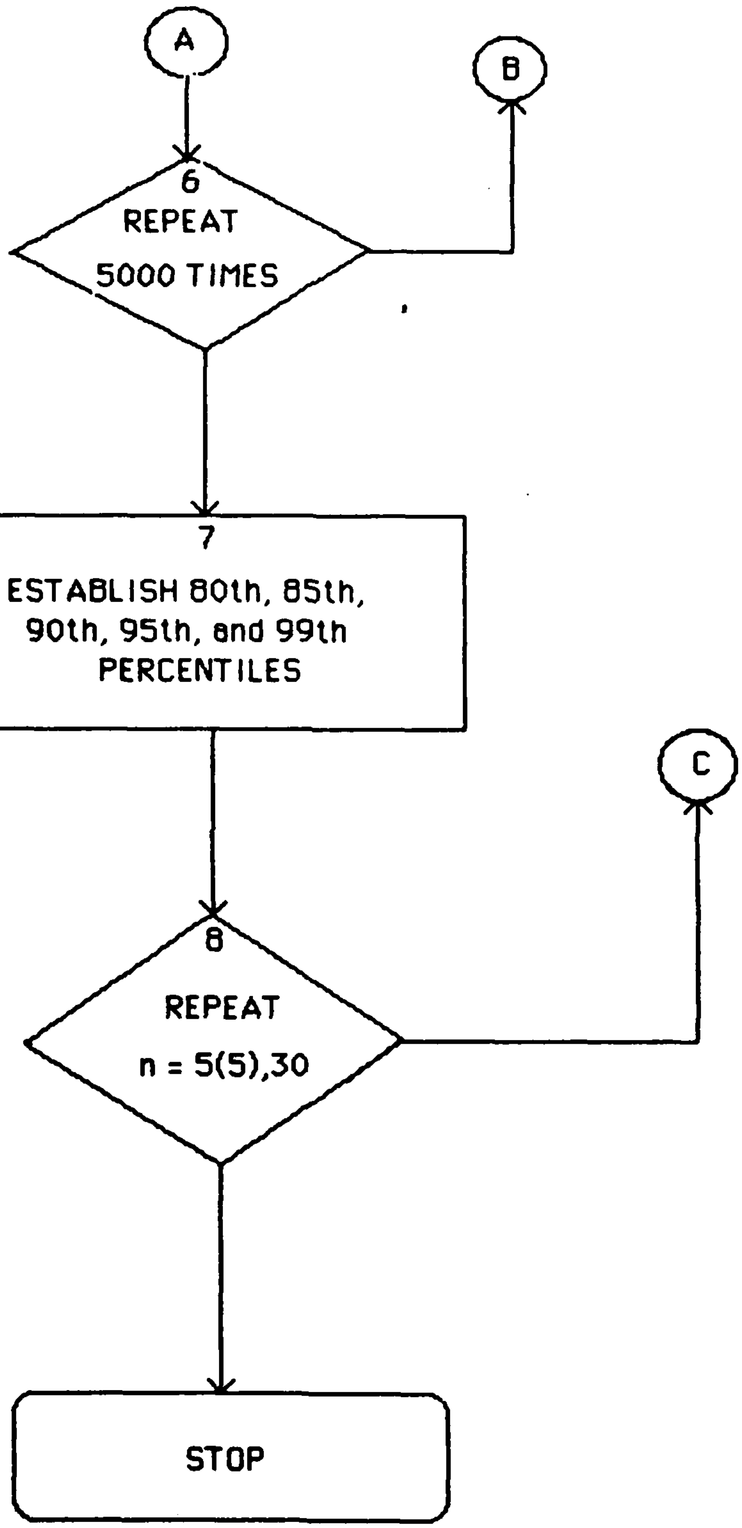

Fig. 2 (Cont'd.) Critical Value Flowchart

was used to order the deviates.

Step 3: Estimating the Parameters. As mentioned in Chapter III, the MLE was used to estimate the location and scale parameters. The actual computer program for the MLE was derived from a program included in a text by D. F. Andrews and P. J. Bickel (4:17).

Step 4: Generate the Hypothesized Distribution Function F(x). With the estimated location and scale parameters from Step 3 and the ordered deviates from Step 2, equation (12) yields the hypothesized CDF.

Step 5: Calculate the Modified KS, AD, and CVM test statistics. Equations (6), (8), and (10) are solved using the hypothesized CDF and the ordered random deviates.

Step 6: Repeat 5000 times. Steps 1 - 5 will be repeated 5000 times to generate 5000 independent KS, AD, and CVM statistics.

Step 7: Determine the Critical Values. A bit of background is important here to understand this step. Critical values are important in hypothesis testing since these values are what will be checked to verify H_0. The whole purpose of this thesis is to generate those critical value tables to use when H_0 states that the actual distribution of the data is Cauchy. Since all the values derived up until now are based on Cauchy random deviates, that H_0 is true for our samples.

Steps 1 - 6 have generated 5000 order statistics for each of the GOF tests. Combined with commonly used α levels, where α is the maximum probability of rejecting a true H_0, all that needs to be determined is the point where, in the range of the order statistics, each of the α levels fall. The mirror image of this is to work with the percentiles, or the $1 - \alpha$ levels. These then become the minimum probability of accepting H_0 when true. The points where those levels fall are the critical values.

To get the critical values different techniques are available. A straightforward technique is to select that order statistic which, as a percentage of the total statistics, matches the percentile level, e.g., for the 80th percentile and 5000 order statistics, the critical value would be the 4000th one. This was the technique used by Green and Hegazy (16). Recently, a more precise technique has been developed, that of plotting positions (29:7).

Plotting positions depend on the bootstrap method (13). The technique involves locating the discrete order statistics on a continuous spectrum. This is accomplished by taking the space between the statistics and representing it as a piecewise linear function. With that function, it is possible to interpolate between the discrete values of the statistics and get more accurate critical values (29:7). The interpolation is done by plotting the order statistics against a plotting position which represents the order statistics on a zero to one scale.

There are many different plotting positions, and prior theses have looked at them and did not find any significant difference between them when it comes to calculating the EDF statistics (6;29;38). Harter (19) recently did an extensive analysis of various of the plotting positions. One of his findings was that as samples increased over a sample size of 20, the differences between them for the positions they determined were insignificant. With 5000 independent values for each test statistic, one plotting position for this thesis is justified.

This thesis will use the median rank plotting convention. Harter shows this to be closely approximated by (19:1617):

$$Y_i = (i-0.3)/(n-0.4) \qquad (19)$$

where

$i = 1,\ldots,n$

$n = 5000$

Ream (29:11-23) gives an in-depth illustration on how plotting positions are used to determine critical values, therefore only a brief overview will be done here. The order statistics $X_{(1)}$, $X_{(2)},\ldots, X_{(5000)}$ are plotted along the abscissa axis, while the 5000 plotting position are plotted along the ordinate axis. Both sets of points are assigned to positions 2 to 5001. For the plotting positions, the interval [0,1] is completed by setting the first position

to be $Y_0 = 0$, and the 5002nd position to be $Y_{5001} = 1$. For the order statistics, linear extrapolation is used to determine the first and last entries. The first entry is made by linearly extrapolating from the first and second order statistics, limited by a nonnegativity restriction. The last entry is similarly extrapolated from the last and next to last order statistics. For the purposes of the computer program used to generate these values, an array of 5002 values was used for each axis.

The extrapolation of $X_{(5001)}$ and $X_{(0)}$ uses $Y = mx + b$, the linear slope-intercept formula. The first endpoint is calculated as follows:

$$m = (Y_2 - Y_1)/(X_{(2)} - X_{(1)}) \tag{20}$$

$$b = Y_1 - m X_{(1)} \tag{21}$$

$$X_{(0)} = -b/m \tag{22}$$

Since a negative value is not allowed, the minimum value for $X_{(0)}$ is 0 leading to:

$$X_{(0)} = \max(0, -b/m) \tag{23}$$

Similarly, the value for $X_{(5001)}$ can be found.

If straight lines are used between all the 5002 points, a piecewise linear function is produced. At this point linear interpolation can be used to determine any value that might fall between any two consecutive points, necessary in order to calculate the critical values. For example, to find the 85th percentile, the largest plotting position, Y_i ,less than .85 is found. Then the corresponding $X_{(i)}$, along with $X_{(i+1)}$ and Y_{i+1} are used to linearly interpolate the critical value using:

$$m = (Y_{i+1} - Y_i)/(X_{(i+1)} - X_{(i)}) \tag{24}$$

$$b = Y_i - m X_{(i)} \tag{25}$$

$$\text{Critical Value} = (p - b)/m \tag{26}$$

The critical value percentiles used for this thesis were 80, 85, 90, 95, 99.

Step 8: Repeat steps 1-7 for each of sample sizes 5,10,15,20,25, and 30. These sample sizes have been used in developing critical value tables for other distributions (6;27).

The resulting critical value tables are in Appendix A.

The Power Study

Once the critical value tables are generated, this thesis then compares the power of the three test statistics against several alternate distributions. As mentioned previously, the concept of power is important when using EDF statistics, since the intent with the hypothesis testing is to accept the null hypothesis. At the same time, one wants to feel confident that the alternate hypothesis is false. By having a comparison of the power of the three statistics, someone testing for the Cauchy distribution can select the test which best protects against likely alternate distributions.

The alternate distributions used for this thesis are the Weibull, with shape parameter of 3.5, the Gamma, with shape parameter of 2.0, the Beta, with the P and Q parameters of 2 and 3, respectively, the Exponential, with the shape parameter of 2.0, the Normal, and the Double Exponential, with the shape parameter of 2.0.

The logic of the power study basically follows that of the critical value table generation, except that instead of starting with Cauchy random deviates, deviates from the above named distributions are used. Since the program to accomplish the power study is simpler and less time consuming when run, the number of statistics calculated for each distribution was set at 10,000 instead of the 5,000 used for the critical value tables.

The first step in the power study involved generating random deviates

for the alternate distributions using IMSL subroutines. Then, since the null hypothesis is that the underlying distribution is Cauchy, steps 2,3,4, and 5 of Fig.2 were performed. This involved ordering the data, estimating the parameters, computing the hypothesized F(x), and calculating the test statistics. These statistics are then compared with the critical values generated for each respective test for alphas of .05 and .01. A counter is incremented each time the calculated test statistic exceeds the critical value. This tracks how many times the null hypothesis is correctly rejected. Then, the total number of rejections is divided by 10,000 to obtain the power. This is repeated for each of the alternate distributions, and finally for each of the different sample sizes (5, 15, 25, 50). The resulting power comparison tables are in Appendix B.

V. Use of the Tables

Chapter Overview

This chapter will discuss the basic procedure involved in using the critical value tables generated in this thesis.

Use of the Tables

The critical value tables will be used to determine whether or not to accept the null hypothesis, that the distribution of the sample data points, $F_n(x)$, is the Cauchy distribution, $F(x)$. The appropriate statistic is calculated using equation (6), (8), or (10). The calculated statistic is compared to the critical value in the tables (for a given n and α) and if the statistic value is greater than the critical value, the null hypothesis is rejected.

The following steps are used in the above analysis (6:28-29):

1. The user will select the appropriate α-level and sample size. As stated earlier, α sets the maximum probability of rejecting the null hypothesis when it is true.

2. Select n random observations from the total population. Order these observations from the smallest to the largest.

3. Estimate the location and scale parameters for the sample. The

estimator must be invariant for the results to be meaningful.

4. Specify the Cauchy distribution using the above estimated location and scale parameter.

5. Calculate the test statistic of interest -- KS, CVM or AD. This can be done using Eq (6), (8), or (10), respectively.

6. Given the n and α, look up the critical value from the tables in Appendix A.

7. If the test statistic value is greater than the critical value, then the null hypothesis is rejected. However, if the critical value is less than or equal to the test statistic value, then there is a failure to reject the null hypothesis. The conclusion is that there is insufficient evidence to reject the null hypothesis.

VI. Results

Chapter Overview

This chapter discusses the results of this thesis -- the critical value tables and the power tables.

Critical Value Tables

The critical value tables for the modified KS, CVM, and AD tests are in Appendix A. They are organized by alpha level (.20, .15, .10, .05, .01) and sample size (5,10,15,20,25,30,50).

The KS critical values all decrease as n increases and α increases. The rate of decrease slows down with increasing n and α. It could be that if n were increased to 40 or 50 the critical values would stabilize at some value. The CVM and the AD critical values also decrease, but only as the α-level increases, while holding n constant. With α constant, there is little change with changing n, in fact, the statistics stay very close together, with slight fluctuation.

Since the critical values are generated through a Monte Carlo process, there is a degree of variability introduced. The error of a Monte Carlo process is proportional to $1/(N)^{1/2}$, with N being the number of iterations of the

simulation (6:33). Therefore, by running the simulation with $N = 10000$ rather than 5000, some of the patterns seen could change. However, due to the greatly increased computer time needed to go from 5000 to 10000, that option was not possible for this thesis.

Power Tables

The first 'alternate' distribution used in the power study was the Cauchy. This was done to validate the values generated in the first part of the thesis. To be valid, the power would have to be close to the α-levels, and that is the case. The powers are not exactly equal to the α-levels, but that is a result of the variability in the Monte Carlo process, as mentioned above.

Among the three tests, the KS is consistently more powerful, across all n and α. There are only 3 or 4 instances where it comes in second and then only in the third significant digit. This could be a result of the KS being fairly insensitive to discrepencies in the tails. The Cauchy has longer and flatter tails and the KS might be deemphasizing the difference there.

Analyzing by sample size shows very poor power at $n = 5$ (.085 being the highest), with power increasing as n increases. When one gets to a sample size of 50, three distributions, the Exponential, the Gamma, and the Beta, have powrs above 0.5 (1.0, .947, .586 respectively). This makes sense as the amount of information available increases with sample size.

When α-levels are analyzed, for $\alpha = .01$, the only reasonable power is

against the Exponential and the Gamma, with a power of .991 and .719 respectively. The next best is only .176. As α increases to .05, power increases across the board, getting up to 1.0 for the Exponential. When looking at the alternate distributions, the distributions with reasonable power are the Exponential, the Gamma, and the Beta. The highest power among all the other distributions is .259, not enough to instill any confidence in the results of the GOF test.

Given the above analysis, if the Cauchy is the distribution in the null hypothesis, one should try for a sample size of 50 or better, and if that is not possible, accept an α of .05 or greater.

VII. Conclusions and Recommendations

Chapter Overview

This chapter gives the conclusions reached in this thesis, and the recommendations made for further study.

Conclusions

1. The critical value tables generated for the Kolmogorov-Smirnov, the Cramer-von Mises, and the Anderson-Darling goodness-of-fit tests for the Cauchy distribution are valid. By using the Cauchy as one of the 'alternate' distributions when doing the power study, the values were validated.

2. Regarding the choice of a test, if the alternate hypothesis is the Exponential, the Gamma, or the Beta, and the sample size is greater than five, then all three of the tests are fairly powerful. However, the Kolmogorov-Smirnov test is the most powerful of the three in any situation.

3. After analyzing the power test, there is a good deal of power available against the Exponential, the Gamma, and the Beta.

Recommendations

1. If possible, the critical value tables should be redone with 10,000 statistics. This would reduce the variability due to the Monte Carlo process,

and would make one more certain of the patterns evident in the tables.

2. With the improvement in power evidenced in the power tables, further power studies should be attempted with larger sample sizes and α-levels. The goal should be to find what combinations would increase the power against the weaker distributions (Weibull, Normal, and Double Exponential).

3. Other distributions should be investigated in further power studies to determine if there are other distributions against which the GOF tests are also powerful.

APPENDIX A

This includes critical value tables for the Kolmogorov-Smirnov, the Cramer-von Mises, and the Anderson-Darling Tests.

TABLE I

CRITICAL VALUES FOR THE MODIFIED KS TEST

ALPHA	N	CRITICAL VALUE
.20	5	0.2898729
.20	10	0.2089256
.20	15	0.1745039
.20	20	0.1542405
.20	25	0.1385744
.20	30	0.1271362
.20	50	0.6989140
.15	5	0.3054360
.15	10	0.2196148
.15	15	0.1839846
.15	20	0.1619931
.15	25	0.1453200
.15	30	0.1338064
.15	50	0.1035580
.10	5	0.3252105
.10	10	0.2335990
.10	15	0.1960747
.10	20	0.1727150
.10	25	0.1548475
.10	30	0.1435490
.10	50	0.1099358
.05	5	0.3480030
.05	10	0.2544265
.05	15	0.2142230
.05	20	0.1878866
.05	25	0.1698837
.05	30	0.1564334
.05	50	0.1200329
.01	5	0.3840281
.01	10	0.2967503
.01	15	0.2463341
.01	20	0.2202671
.01	25	0.2011247
.01	30	0.1826919
.01	50	0.1418185

TABLE II

CRITICAL VALUES FOR THE MODIFIED CVM TEST

ALPHA	N	CRITICAL VALUE
.20	5	0.0971847
.20	10	0.0910045
.20	15	0.0914536
.20	20	0.0930957
.20	25	0.0918603
.20	30	0.0925299
.20	50	0.0911952
.15	5	0.1148650
.15	10	0.1064403
.15	15	0.1068219
.15	20	0.1087681
.15	25	0.1075684
.15	30	0.1090852
.15	50	0.1055738
.10	5	0.1364753
.10	10	0.1280915
.10	15	0.1262927
.10	20	0.1290561
.10	25	0.1304703
.10	30	0.1311479
.10	50	0.1266934
.05	5	0.1668984
.05	10	0.1643178
.05	15	0.1663567
.05	20	0.1694394
.05	25	0.1711696
.05	30	0.1705763
.05	50	0.1629795
.01	5	0.2162196
.01	10	0.2393059
.01	15	0.2558281
.01	20	0.2500614
.01	25	0.2658464
.01	30	0.2640329
.01	50	0.2547385

TABLE III

CRITICAL VALUES FOR THE MODIFIED AD TEST

ALPHA	N	CRITICAL VALUE
.20	5	0.7511249
.20	10	0.7006519
.20	15	0.7101663
.20	20	0.7105355
.20	25	0.6993738
.20	30	0.7045876
.20	50	0.6954132
.15	5	0.8839264
.15	10	0.8087849
.15	15	0.8106228
.15	20	0.8196878
.15	25	0.7957106
.15	30	0.8138990
.15	50	0.7880611
.10	5	1.[illegible]499634
.10	10	0.9686199
.10	15	0.9728056
.10	20	0.9603971
.10	25	0.9719902
.10	30	0.9705017
.10	50	0.9372754
.05	5	1.3589739
.05	10	1.2246506
.05	15	1.2866855
.05	20	1.2586166
.05	25	1.2332463
.05	30	1.2498085
.05	50	1.1945103
.01	5	2.1669748
.01	10	1.8335395
.01	15	1.9464778
.01	20	1.8302892
.01	25	1.9715600
.01	30	1.9268377
.01	50	1.8649824

APPENDIX B

This is the result of the power study.

TABLE IV

POWER TEST FOR THE CAUCHY DISTRIBUTION

LEVEL OF SIGNIFICANCE = .05

N	TEST	ALTERNATE DISTRIBUTIONS CAUCH	WEIBL SH=3.5	GAMMA SH=2.	BETA P2Q3	EXPON SH=2.	NORML	DBLEXP SH=2.
5	K-S	0.051	0.040	0.053	0.042	0.085	0.039	0.007
5	CVM	0.051	0.035	0.047	0.036	0.079	0.033	0.005
5	A-D	0.054	0.029	0.041	0.030	0.068	0.026	0.004
15	K-S	0.051	0.054	0.178	0.084	0.423	0.050	0.011
15	CVM	0.052	0.032	0.149	0.054	0.354	0.029	0.004
15	A-D	0.045	0.015	0.095	0.029	0.259	0.015	0.001
25	K-S	0.050	0.069	0.415	0.139	0.805	0.060	0.020
25	CVM	0.047	0.036	0.285	0.078	0.619	0.032	0.010
25	A-D	0.049	0.028	0.254	0.070	0.586	0.026	0.010
50	K-S	0.054	0.225	0.947	0.586	1.000	0.184	0.259
50	CVM	0.051	0.115	0.724	0.287	0.962	0.097	0.193
50	A-D	0.051	0.213	0.789	0.478	0.976	0.170	0.313

LEVEL OF SIGNIFICANCE = .01

N	TEST	ALTERNATE DISTRIBUTIONS CAUCH	WEIBL SH=3.5	GAMMA SH=2.	BETA P2Q3	EXPON SH=2.	NORML	DBLEXP SH=2.
5	K-S	0.008	0.005	0.006	0.005	0.011	0.004	0.000
5	CVM	0.010	0.005	0.006	0.006	0.013	0.005	0.000
5	A-D	0.010	0.004	0.006	0.004	0.011	0.004	0.000
15	K-S	0.015	0.011	0.060	0.023	0.199	0.011	0.001
15	CVM	0.010	0.003	0.036	0.009	0.134	0.004	0.000
15	A-D	0.010	0.001	0.017	0.003	0.084	0.002	0.000
25	K-S	0.009	0.010	0.125	0.025	0.489	0.010	0.001
25	CVM	0.009	0.004	0.089	0.012	0.330	0.003	0.000
25	A-D	0.008	0.002	0.055	0.005	0.242	0.001	0.000
50	K-S	0.012	0.036	0.719	0.176	0.991	0.033	0.032
50	CVM	0.013	0.012	0.393	0.052	0.828	0.011	0.008
50	A-D	0.012	0.012	0.375	0.058	0.816	0.010	0.012

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ABSTRACT

The Kolmogorov-Smirnov, Anderson-Darling, and Cramer-von Mises critical values are generated for the Cauchy distribution. The critical values are used for testing the null hypothesis that a set of observations follow a Cauchy distribution when the location and scale parameters are unknown and estimated from the sample. A Monte Carlo simulation, using 5000 repetitions, was used to generate the critical values for sample sizes of 5(5)30 and 50. goodness of fit

A power study was performed using Monte Carlo simulation for the Kolmogorov-Smirnov, Anderson-Darling, and Cramer-von Mises tests. Sample sizes of 5, 15, 25 and 50 were used for six alternate distributions, for alpha levels of .05 and .01. Analyzing by sample size shows very poor power for a sample size of five. As the sample size increases so does the power, so that at a sample size of fifty, the powers against three of the six distributions is .5 or better. Among the three tests, the Kolmogorov-Smirnov is consistently more powerful, regardless of sample size or alpha level. Keywords: theses; hypothesis testing.

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